Improving top-K recommendation with truster and trustee relationship in user trust network

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Goal of Recommender System

items

users

	A	B	C	D
A	5	?	?	3
B	4	?	?	2
C	?	1	3	1

Probabilistic Matrix Factorization (PMF)

- Ratings can be approximated from probabilistic methods.

1. Modeling rating variables

$$p(R|U,V,\sigma^2)=\prod_{i=1}^{N}\prod_{j=1}^{M}\left[\mathcal{N}(R_{ij}|U_i^TV_j,\sigma^2)\right]^{I_{ij}}$$

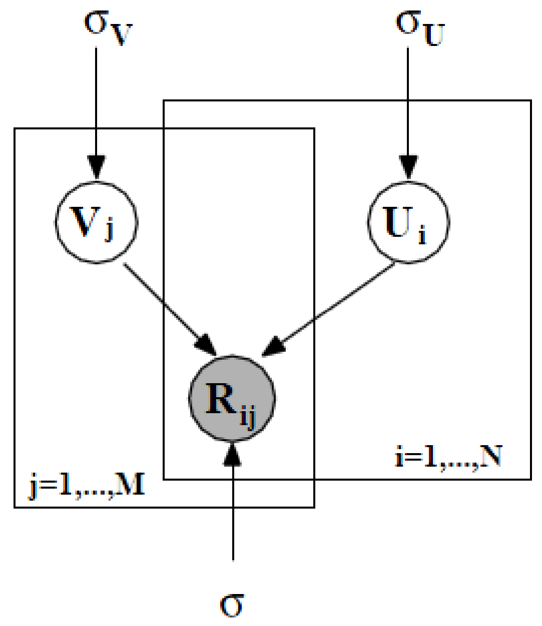
2. Modeling user and item variables

$$p(U|\sigma_U^2)=\prod_{i=1}^{N}\mathcal{N}(U_i|0,\sigma_U^2\mathbf{I}), \qquad p(V|\sigma_V^2)=\prod_{j=1}^{M}\mathcal{N}(V_j|0,\sigma_V^2\mathbf{I}).$$

3. Posterior probability over user and item variables

$$p(U,V|R,\sigma,\sigma_U,\sigma_V)\propto p(R|U,V,\sigma)p(U\big|\sigma_U^2)p(V|\sigma_V^2)$$

$$\begin{aligned}\ln p(U,V|R,\sigma^2,\sigma_V^2,\sigma_U^2) = &-\frac{1}{2\sigma^2}\sum_{i=1}^{N}\sum_{j=1}^{M}I_{ij}(R_{ij}-U_i^TV_j)^2-\frac{1}{2\sigma_U^2}\sum_{i=1}^{N}U_i^TU_i-\frac{1}{2\sigma_V^2}\sum_{j=1}^{M}V_j^TV_j \\ &-\frac{1}{2}\left(\left(\sum_{i=1}^{N}\sum_{j=1}^{M}I_{ij}\right)\ln\sigma^2+ND\ln\sigma_U^2+MD\ln\sigma_V^2\right)+C, \qquad (3)\end{aligned}$$



<The graphical model of PMF>

Limitation of PMF

- They suffer from data sparsity problem
- What is *“Data sparsity problem”*?
  - Recommendation is hardly accurate due to lack of observations (i.e., ratings)
- To tackle the data sparsity problem, incorporating auxiliary information becomes important
  - Time related information
  - Textual data
  - **Social network relationships among users (Focus of this work)**

Motivation

- Most existing works exploit social information to reduce the rating prediction error, e.g., RMSE
- However, users are interested in seeing a list of top-k items rather than predicted ratings

	A	B	C	D	E
User1	5	?	?	3	?

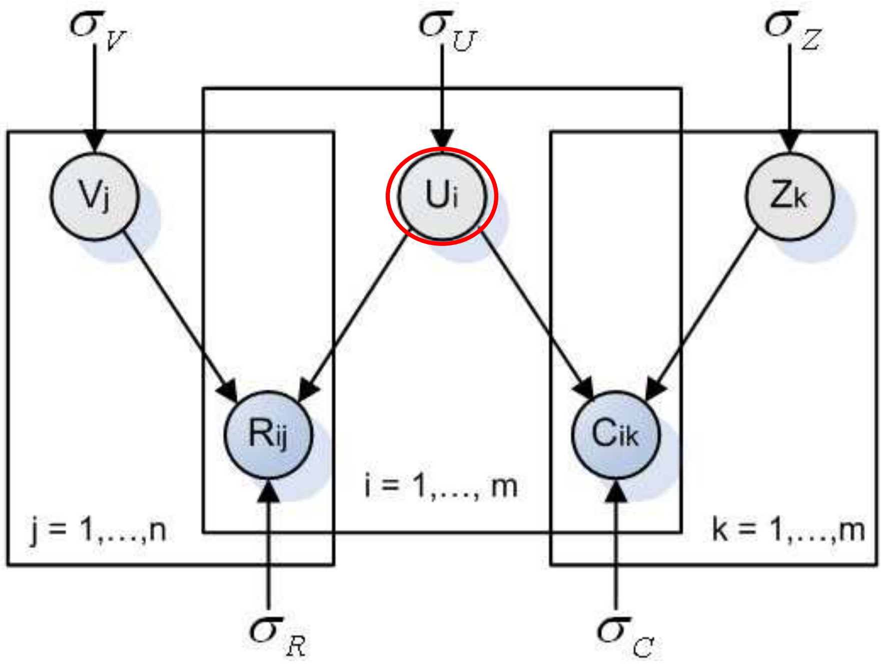
User1
Rank1: C
Rank2: E
Rank3: B

- Minimizing the rating prediction error does not always result in a better top-k list of items [Cremonesi et al. RecSys 2010]
- **Therefore, let’s focus on improving the result of top-k list of items using social network information.**

Related Work

Related Work 1 – Social Recommender System

1. SoRec [Ma et al., CIKM 09]
   - Matrix Co-Factorization technique

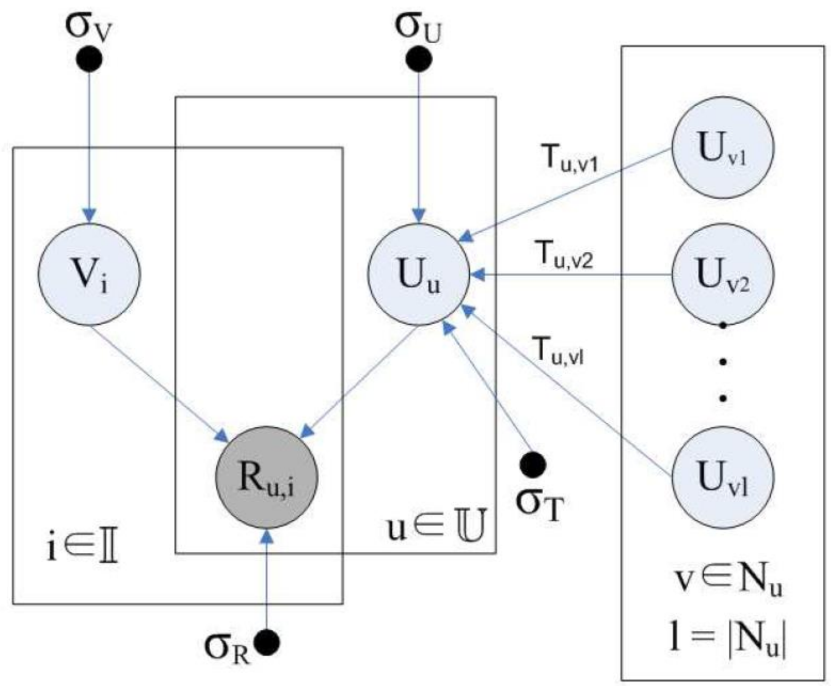


- Loss function

$$\mathcal{L}(R,C,U,V,Z) = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n} I_{ij}^{R}(r_{ij} - g(U_i^T V_j))^2 + \frac{\lambda_C}{2}\sum_{i=1}^{m}\sum_{k=1}^{m} I_{ik}^{C}(c_{ik}^{*} - g(U_i^T Z_k))^2 + \frac{\lambda_U}{2}\|U\|_F^2 + \frac{\lambda_V}{2}\|V\|_F^2 + \frac{\lambda_Z}{2}\|Z\|_F^2, \quad (9)$$

Related Work 1 – Social Recommender System

2. SocialMF [Jamali et al., RecSys 10]
    - Models trust propagation



- Loss function

$$\mathcal{L}(R,T,U,V) = \frac{1}{2}\sum_{u=1}^{N}\sum_{i=1}^{M} I^{R}_{u,i}(R_{u,i} - g(U_u^T V_i))^2 + \frac{\lambda_U}{2}\sum_{u=1}^{N} U_u^T U_u + \frac{\lambda_V}{2}\sum_{i=1}^{M} V_i^T V_i + \frac{\lambda_T}{2}\sum_{u=1}^{N}\Big((U_u - \sum_{v\in N_u} T_{u,v}U_v)^T (U_u - \sum_{v\in N_u} T_{u,v}U_v)\Big)$$

Related Work 1 – Social Recommender System

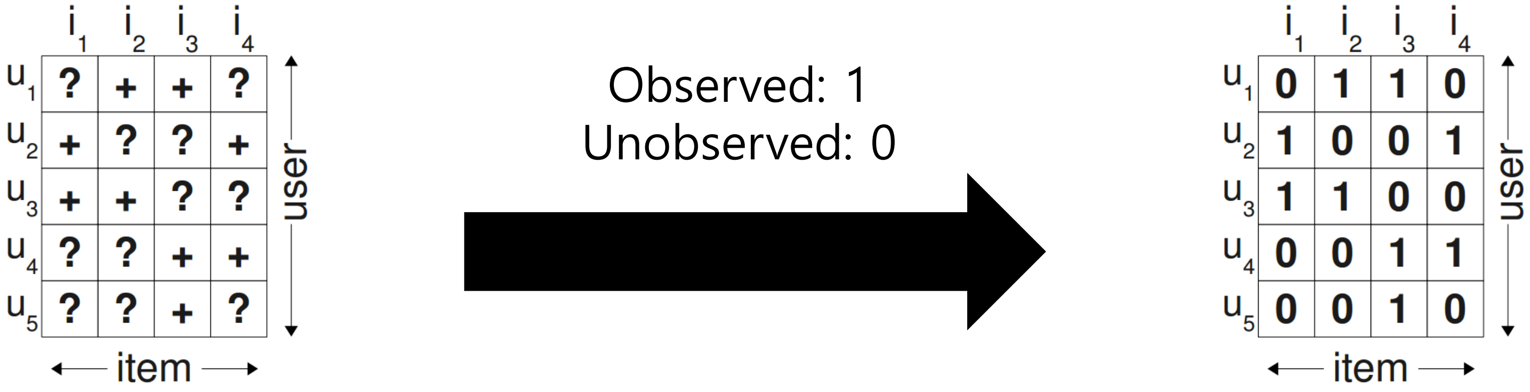
- Aforementioned social recommender systems mainly focus on minimizing the rating prediction error, e.g., MAE, RMSE
- Improving MAE and RMSE does not lead to improving top-k performance
- **Therefore, we focus on finding a better top-k list of items**

Related Work 2 – Top-k ranking RS

- Several approaches have been proposed for top-k recommendation
    - Can be cast as Learning-to-Rank (LTR) problem
- Learning-to-Rank
    - A Supervised ML method that directly builds a ranking list from training data
    - Pair-wise models
        - Learn users' relative preferences of each item pair
    - List-wise models
        - Directly predicts ranking list of items for each user based on the distance between the ground truth ranking list and the predicted list.

Related Work 2 – Top-k ranking RS

1. BPR [Rendle et al., UAI 09] (Pairwise method)

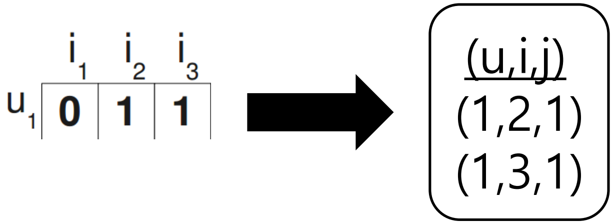


	i_1	i_2	i_3	i_4
u_1	?	+	+	?
u_2	+	?	?	+
u_3	+	+	?	?
u_4	?	?	+	+
u_5	?	?	+	?

Observed: 1
Unobserved: 0

	i_1	i_2	i_3	i_4
u_1	0	1	1	0
u_2	1	0	0	1
u_3	1	1	0	0
u_4	0	0	1	1
u_5	0	0	1	0

$$\mathrm{BPR}(\mathcal{D}_S) = \underset{\Theta}{\operatorname{argmax}} \sum_{(u,i,j)\in\mathcal{D}_S} \ln \sigma(\hat{s}_{u,i}(\Theta) - \hat{s}_{u,j}(\Theta)) - \lambda\|\Theta\|^2$$



	i_1	i_2	i_3
u_1	0	1	1

(u,i,j)
(1,2,1)
(1,3,1)

i: positive item
j: negative item

- ▶ $\mathcal{D}_S$ contains all pairs of positive and negative items for each user,
- ▶ $\hat{s}_{u,i}(\Theta)$ is the predicted score for user u and item i

Related Work 2 – Top-k ranking RS

2. ListRank [Shi et al., RecSys 10] (Listwise method)

- List-wise learning-to-rank algorithm + Matrix Factorization
- Top-one probability $P_{l_i}(R_{ij}) = \frac{\varphi(R_{ij})}{\sum_{k=1}^{K} \varphi(R_{ik})}$

Computes the probability of an item scored R_{ij} being ranked in top-1 position

- Loss function

$$L(U,V) = \sum_{i=1}^{M} \left\{ -\sum_{j=1}^{N} \overbrace{P_{l_i}(R_{ij}) \log P_{l_i}\left(g\left(U_i^T V_j\right)\right)}^{\text{Cross entropy}} \right\} + \frac{\lambda}{2}\left(\|U\|_F^2 + \|V\|_F^2\right)$$

$$= \sum_{i=1}^{M} \left\{ -\sum_{j=1}^{N} I_{ij} \frac{\exp(R_{ij})}{\sum_{k=1}^{N} I_{ik} \exp(R_{ik})} \log \frac{\exp\left(g\left(U_i^T V_j\right)\right)}{\sum_{k=1}^{N} I_{ik} \exp\left(g\left(U_i^T V_j\right)\right)} \right\} + \frac{\lambda}{2}\left(\|U\|_F^2 + \|V\|_F^2\right)$$

Preliminary: Defining permutation probability

- Probability of a permutation is defined with Plackett–Luce model

$$P(\pi \mid f) = \prod_{j=1}^{m} \frac{\varphi(f(x_{\pi(j)}))}{\sum_{k=j}^{m} \varphi(f(x_{\pi(k)}))}$$

$$P_{l_i}\left(R_{ij}\right) = \frac{\varphi\left(R_{ij}\right)}{\sum_{k=1}^{K} \varphi\left(R_{ik}\right)}$$

  - Example

$$P(\mathrm{ABC} \mid f) = \boxed{\frac{\varphi(f(\mathrm{A}))}{\varphi(f(\mathrm{A})) + \varphi(f(\mathrm{B})) + \varphi(f(\mathrm{A}))}} \cdot \boxed{\frac{\varphi(f(\mathrm{B}))}{\varphi(f(\mathrm{B})) + \varphi(f(\mathrm{C}))}} \cdot \boxed{\frac{\varphi(f(\mathrm{C}))}{\varphi(f(\mathrm{C}))}}$$

P(A ranked No.1)

P(B ranked No.2 | A ranked No.1)
= P(B ranked No.1)/(1- P(A ranked No.1))

P(C ranked No.3 | A ranked No.1, B ranked No.2)

Related Work 2 – Top-k ranking RS

- Although pair-wise models have shown substantial improvements in terms of top-k recommendation, they have issues with high computational complexity
- In this work, we adopt the **list-wise approach**

Related Work 3 –Top-k ranking **Social** RS

1. Sorank: Incorporating social information into learning to rank models for recommendation [Yao et al., WWW 2014]
   - Linearly combine a user's taste and her direct friends' taste

2. SBPR [Zhao et al., CIKM 14]
   - Social network integrated version of BPR [Rendle et al., UAI 09]
   - Optimize the top-k recommendation from relative ordering that can be extracted from purchase history or browsing history

Related Work 3 –Top-k ranking **Social** RS

- SoRank [Yao et al., WWW 2014] does not utilize other important information hidden in social network such as the structural information or follower-followee relationship

- SBPR [Zhao et al., CIKM 14] cannot handle numerical ratings directly

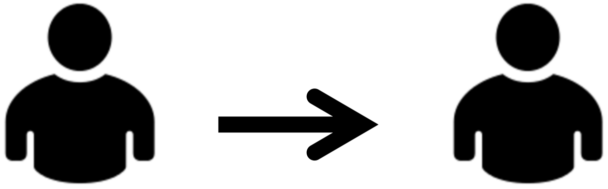
Proposed Method

Problem Definition

- $U = \{u_1, u_2, \dots, u_N\}$: Set of users
- $V = \{v_1, v_2, \dots, v_M\}$: Set of items
- $R = \left[r_{ij}\right]_{N \times M}$: Rating of u_i on v_j
- $S = [s_{ik}]_{N \times N}$: $s_{ik} = 1$, if u_i follows u_k $(s_{ik} \neq s_{ki})$
- <u>Problem</u>
  - Given: The observed rating matrix R and the trust matrix S
  - Goal: Recommend each user a list of unobserved items considering their personal preferences

Method: Modeling Rating

- Due to the asymmetry property ($s_{ik} \neq s_{ki}$), we map each user into two different latent vectors – **Follower** and **Followee**
- Assumption
  - When "$user\ A$" is given several choices of items, he asks the people he follows for their opinions about the items **(Follower role)**
  - The decision made by the "$user\ A$" will influence the people that follow "$user\ A$" **(Followee role)**



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Method: Modeling Rating

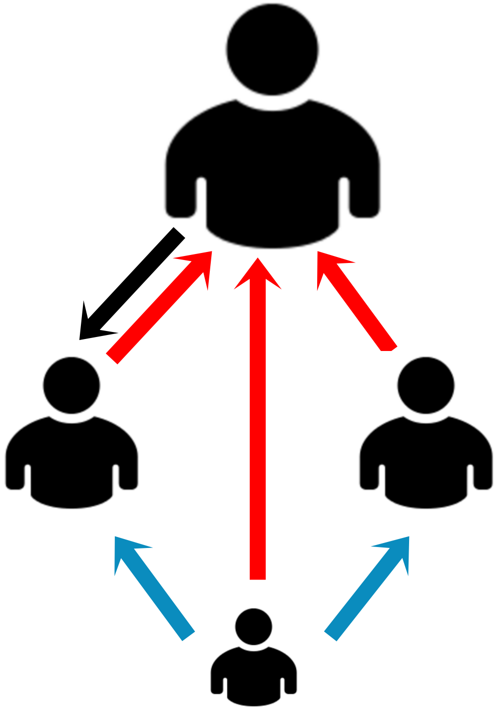
- Rating prediction

Follower + **Followee**

$$\hat{r}_{ij} = g(\mu + b_{u_i} + b_{v_j} + {q_j}^T(\alpha \underbrace{p_i + (1-\alpha) w_i}_{\textbf{User}} + |I_i|^{-\frac{1}{2}} \sum_{t \in I_i} y_t + |T_i|^{-\frac{1}{2}} \sum_{v \in T_i} x_v))$$

- b_{u_i}: User bias
- b_{v_j}: Item bias
- p_i: Follower latent vector
- w_i: Followee latent vector
- y_t: implicit influence of items rated by u_i
- x_v: implicit influence of users followed by u_i
- I_i: Set of items rated by u_i
- T_i: Set of users trusted by u_i

Method: Modeling Trust



- To reflect the structural information of trust network…
  - Adjust s_{ik} based on the degrees of nodes such that
    - Give lower weights to those who *follow* many users
    - Give higher weights to those who *are followed* by many users

$$s_{ik}^{*} = \sqrt{\frac{Indegree(v_k)}{Outdegree(v_i) + Indegree(v_k)}} \times s_{ik}$$

v_i: Node for u_i

Method: Modeling Trust

- Trust prediction

$$\hat{s}_{ik} = g(b_{p_i} + b_{w_k} + w_k^T p_i)$$

- ➢ b_{p_i}: Follower bias
- ➢ b_{w_k}: Followee bias
- ➢ p_i: Follower latent vector
- ➢ w_k: Followee latent vector

Method: Unified Model

- Final Loss function

$$\mathcal{L} = \overset{\text{Rating}}{\boxed{-\sum_{i=1}^{N}\sum_{j\in I_i} P_{l_i}(r_{ij}) \log P_{l_i}(\hat{r}_{ij})}} - \overset{\text{Trust}}{\boxed{\lambda_t \sum_{i=1}^{N}\sum_{k\in T_i} P_{l_i}(s^*_{ik}) \log P_{l_i}(\hat{s}_{ik})}}$$

$$+ \frac{\lambda_p}{2}||p_i||_F^2 + \frac{\lambda_w}{2}||w_i||_F^2 + \frac{\lambda_v}{2}||q_j||_F^2 + \frac{\lambda_c}{2}(\sum_{t\in I_i}||y_t||_F^2 + \sum_{v\in T_i}||x_v||_F^2)$$

$$+ \frac{\lambda_b}{2}(|||b_{u_i}||_F^2 + ||b_{v_j}||_F^2 + |b_{p_i}||_F^2 + ||b_{w_i}||_F^2)$$

Experiments

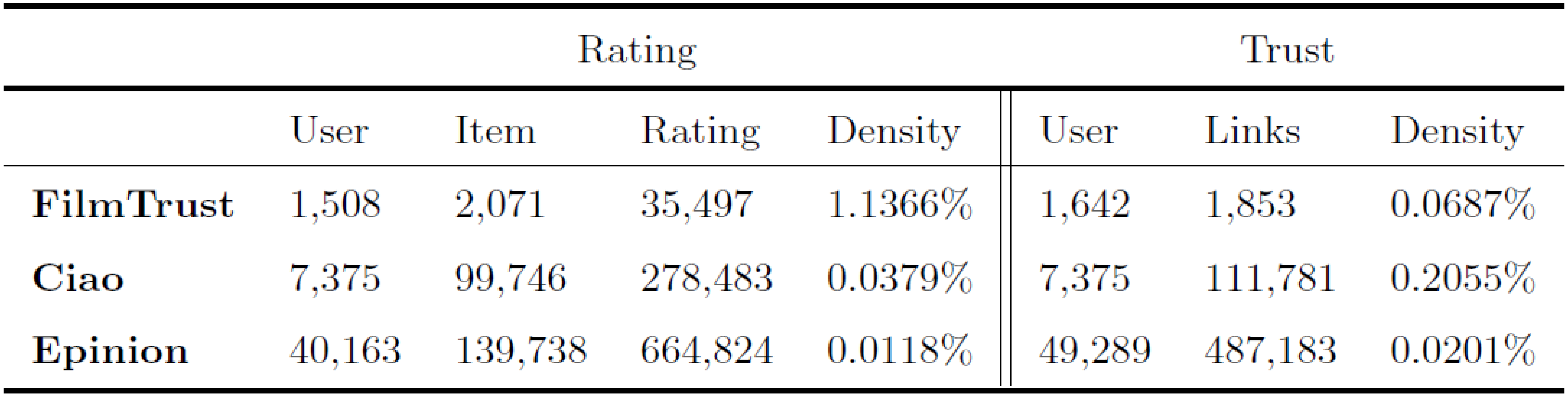
Experiments

- Questions to answer

1. How does TRecSo perform compared with other related competitors?
2. Does considering the social network structure enhance the performance of TRecSo?
3. How does the trade-off parameter of TRecSo affect the quality of top-k recommendation?

Experiments

- Data statistics



	Rating				Trust		
	User	Item	Rating	Density	User	Links	Density
FilmTrust	1,508	2,071	35,497	1.1366%	1,642	1,853	0.0687%
Ciao	7,375	99,746	278,483	0.0379%	7,375	111,781	0.2055%
Epinion	40,163	139,738	664,824	0.0118%	49,289	487,183	0.0201%

Experiments

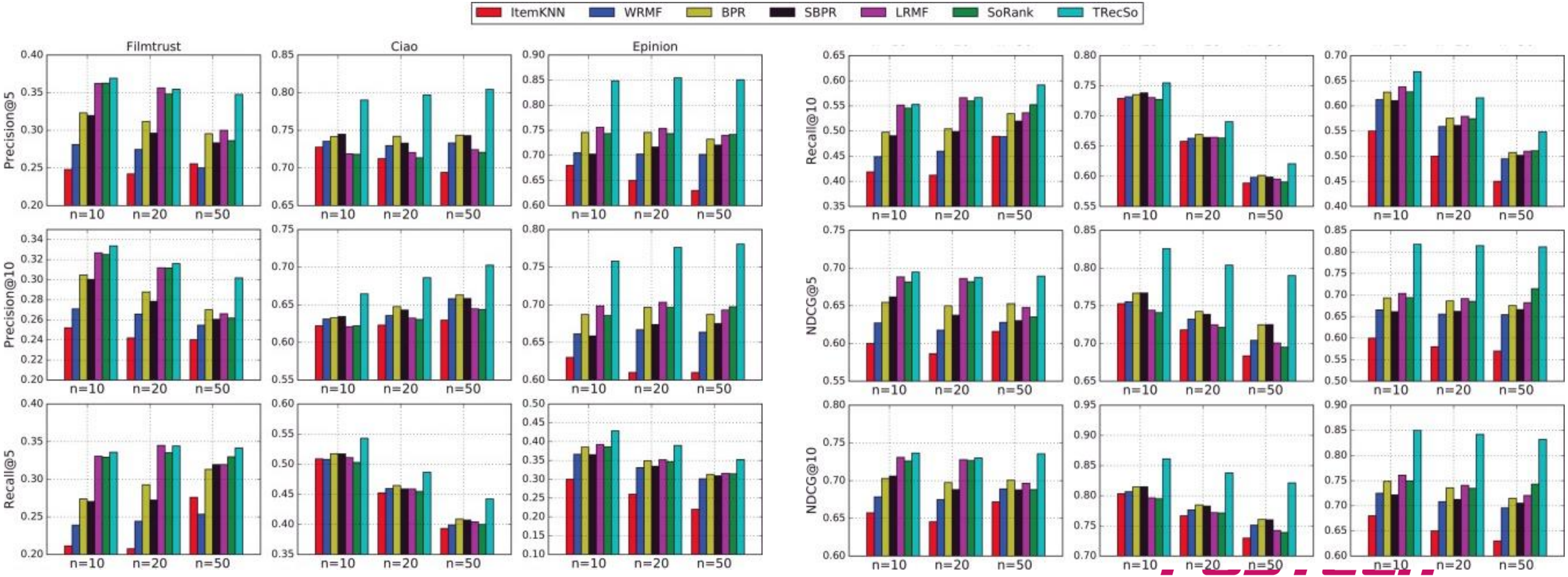
- Experiment protocol: Weak generalization
  - A widely used protocol for evaluating the performance of top-k recommender system
  - Evaluated by predicting the rank of unrated items for users known at training time
- We randomly select N=10, 20, 50 observed ratings for each user for training
- The model performance is tested on the remaining observed ratings

Experiments

- Competitors
  - Traditional CF method
    - ***ItemKNN:*** A traditional recommendation method based on similarity of items
  - Ratings-only-based LTR methods
    - ***WRMF***: A weighted matrix factorization algorithm with implicit feedback data
    - ***BPR***: An item recommendation algorithm based on pair-wise Learning-to-Rank strategy combined with matrix factorization.
    - ***ListRank***: A list-wise Learning-to-Rank method combined with matrix factorization
  - Social network-based LTR methods
    - ***SBPR****: An extended version of BPRMF by including social network information*
    - ***SoRank:*** A social network based list-wise Learning-to-Rank algorithm that linearly combines a users taste and her direct friends tastes in optimizing the top-k recommendation

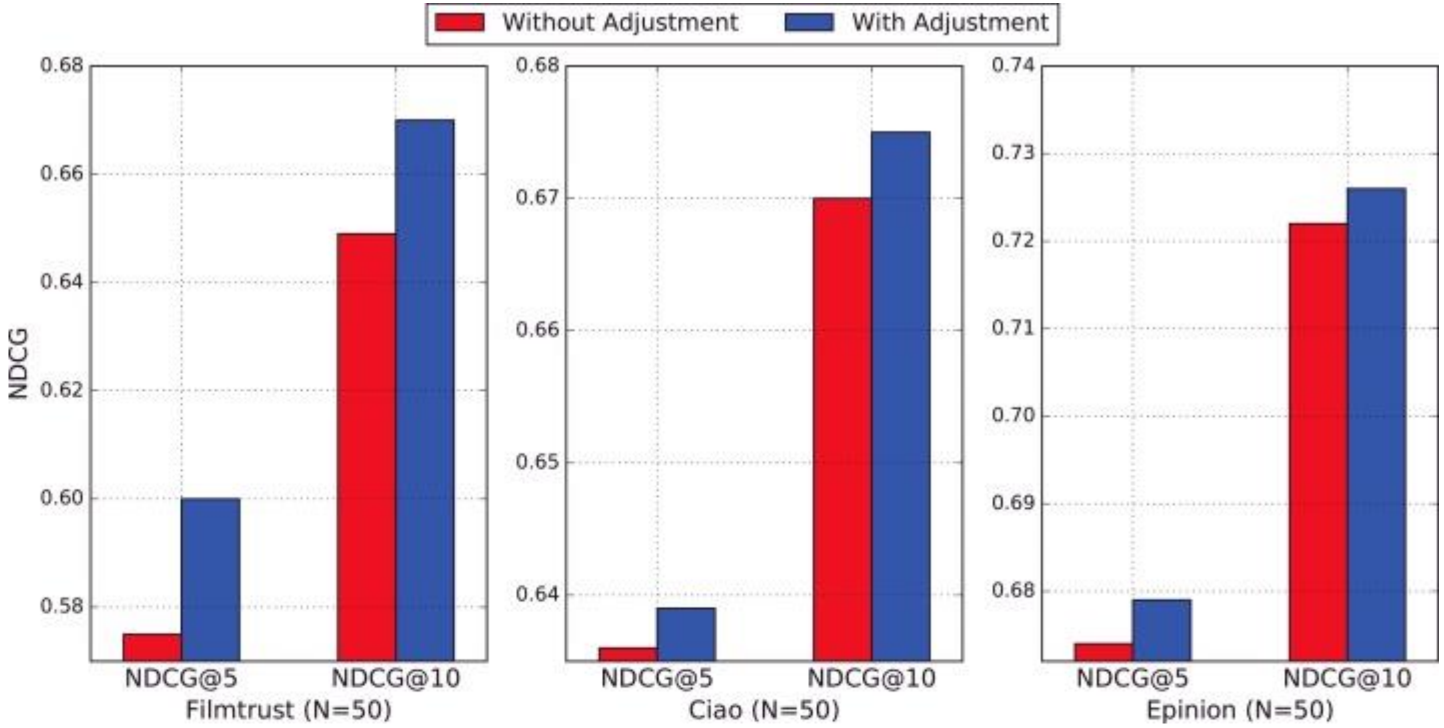
Experiments: Question #1

- Performance comparison



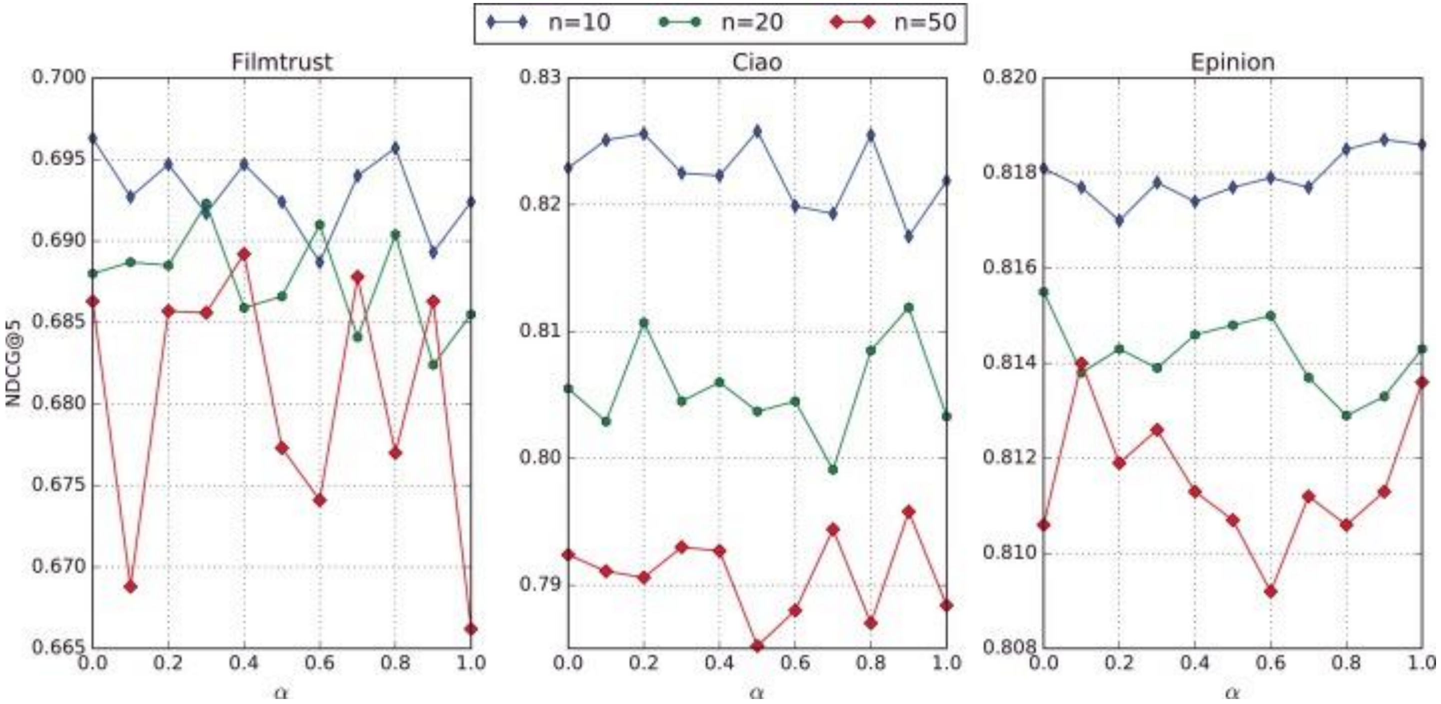
Experiments: Question #2

- Impact of considering graph structural information



Experiments: Question #3

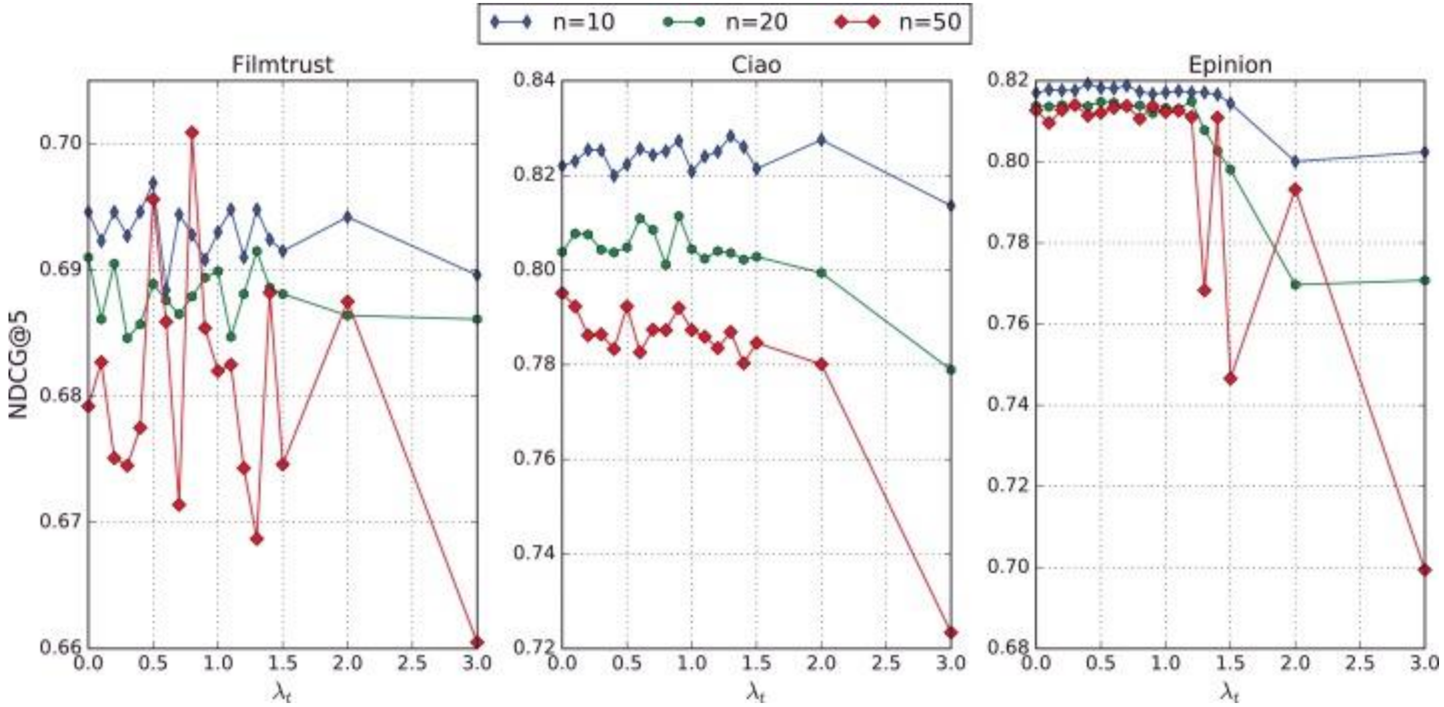
- Impact of trade-off parameters α
  - α: The parameter for balancing the relative importance of influence of follower and followee



- A proper value of α improves the recommendation quality

Experiments: Question #3

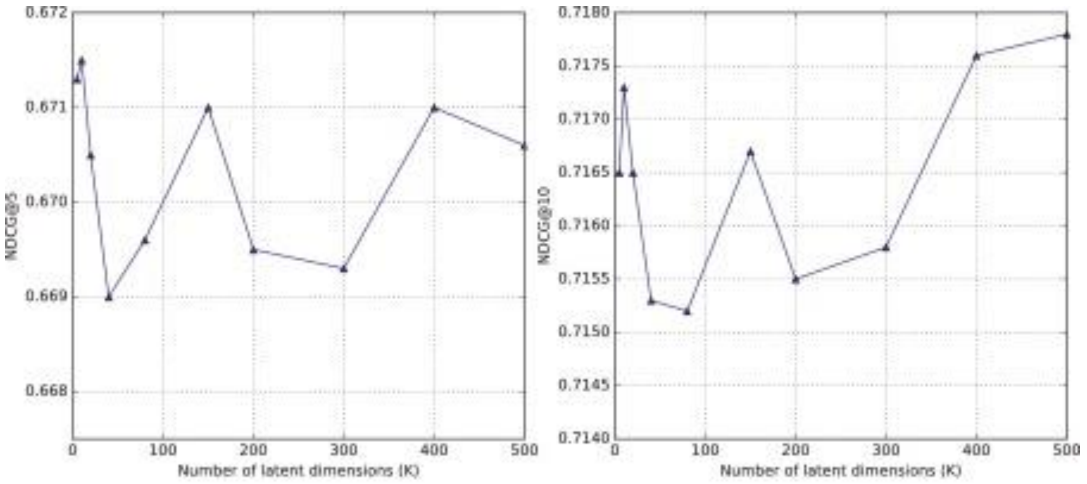
- Impact of trade-off parameters λ_t
  - λ_t: The parameter that controls the importance of trust regularization



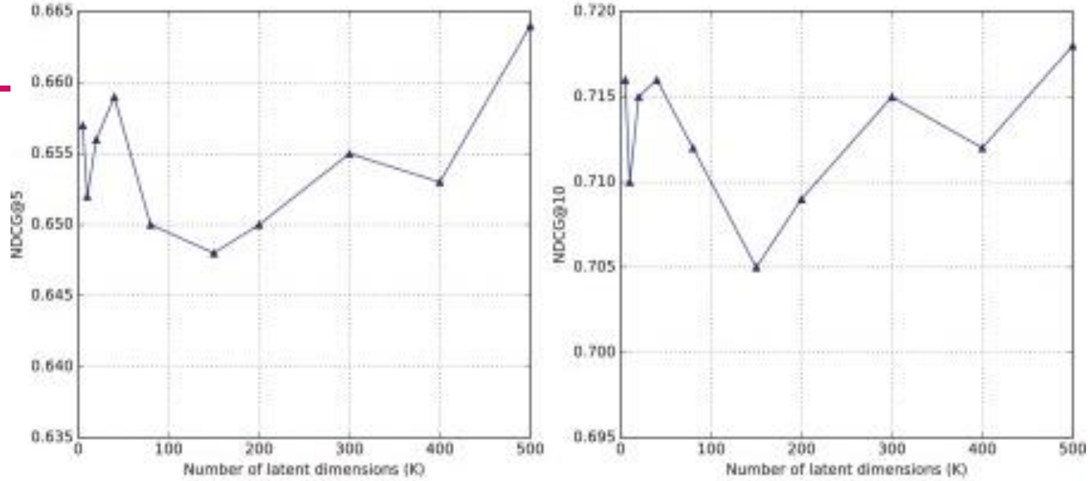
- Incorporating trust information ($\lambda_t > 0$) improves the recommendation

Experiments: Question #4

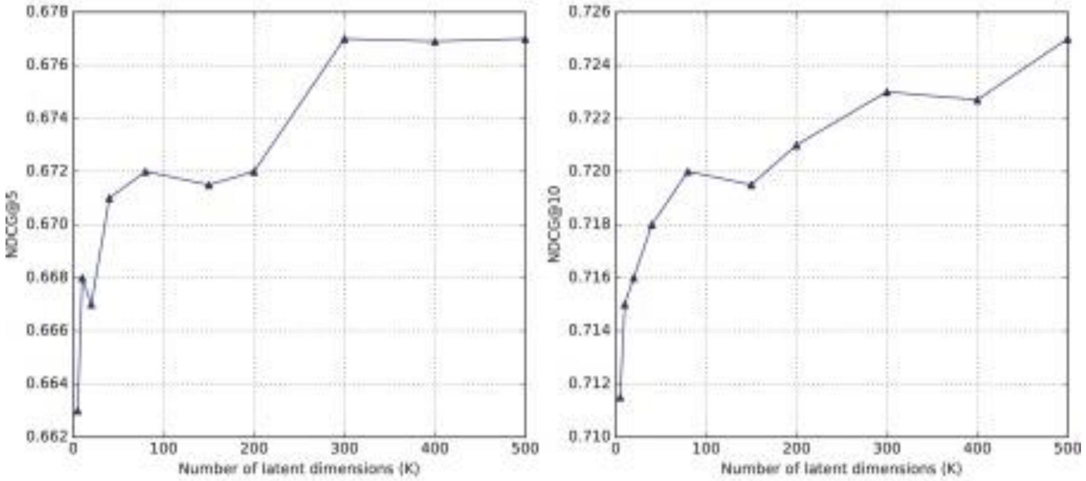
- Dimensionality analysis



(a) Filmtrust dataset (n=20)



(b) Ciao dataset (n=20)



(c) Epinion dataset (n=20)

Experiments: Question #4

- Dimensionality analysis
  - Generally, it is known that the performance of recommendation improves as the number of latent dimensions increases
  - Filmtrust / Ciao → No trend
  - Epinion → Desired trend
  - Reason
    - Each latent dimension represents the profile of user's interest and item's features
    - However, for datasets like Epinion (large number of users and items), the performance of recommendation improves as the number of latent dimensionality increases.
- Trade-off between performance and complexity
  - If the number of dimensions is too large, the complexity will significantly increase
  - Find a proper number of latent dimensions!

Conclusion

- This work proposes a novel MF based recommendation method that optimizes the top-k ranking prediction accuracy
  - Considered two roles of users as follower and followee
  - Considered the trust network information
- TRecSo significantly outperforms the state-of-the-art algorithms in the top-k ranking accuracy of recommendation

Reference

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